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NUMI BPM detectors

The goal.

1. **Continuous monitoring of the beam centroid position.**
2. **Beam intensity measurement.**

- **Specifications**
  - **12 detectors @ 0.2 mm resolution – beam transport**
  - **4 detectors @ 0.05mm resolution – beam targeting**

- **The requirements for the BPM's functional accuracy**

1. **BPM's along the transport line +/- 0.2mm.**
2. **BPM's along the target line-up +/- 0.05mm.**
3. **Beam intensity measurement +/- 3%.**

- **What type BPM will be used?**

Two different sizes, but the same in style of BPM's will bused. For the first part transport line will be used standard BPM's (4 inches in diameter) as in current MI transfer lines. And for the target line-up will use the BPM's 2 .125 inches in diameter. For the last one no experimental data is available.

Both BPM 's have diagonally cut circular cross-section pickup electrode. That electrode shape was chosen to have a linear response to beam displacement.

To define the electronic specifications (which must be reliable and affordable) a simple analytical calculation for was done output signal, noise budget and position sensitivities for that type BPM's.

To be convinced of calculations the calculated values are compared to the measurements of standard BPM.

- **Signal analysis and the main BPM's parameters**

We have made the approximation that the beam acts as a perfect current source in generating image current on the inside surface of the electrodes.

The analysis was done for Gaussian bunch shape:

$$I_b(t) = eN_p / \sigma \sqrt{2\pi} \times \mathrm{Exp}(-t^2/2\sigma^2)$$

where e is the electron charge in Coulombs, N_p is the number of the protons, σ is the RMS bunch temporal length (as bunch length we take $\pm 2\sigma$, that correspond to 95% protons in the bunch).Further analysis will be done in the frequency domain.

Assuming that the bunch is symmetric in time, centered at time $t=0$

and is in a pulse train with the bunch spacing T, the Gaussian bunch shape can be expended in a cosine series with angular frequency $\omega_0 = 2\pi/T$.

$$I_b(t) = \langle I_b \rangle \left[1 + 2\sum_m A_m Cos(m\omega_0 t) \right]$$

where A_m =Exp($-m^2\omega_0^2\sigma^2/2$) is the harmonic amplitude factor and $\langle I_b \rangle$ = eN/T is the average (dc) bunch current.

The image current flowing over the characteristic impedance is equal to the time derivative of the induced charge Q(t) on the electrode

$$i_s(t) = dQ(t)/dt = d\{I_b(t)/\beta_b c \times Z(x, m\omega_0)\}/dt$$

or

$$i_s(t) = 2\sum_m \langle I_b \rangle\, m\omega_0 / \beta_b c \times \mathrm{Exp}(-m^2\omega_0^2\sigma^2/2)\, Z(x, m\omega_0) \times \sin(m\omega_0 t)$$

where q(t)= $I_b(t)/\beta_b c$ is the beam charge density, $Z(x, m\omega_0)$ is the "transfer function" from beam current to BPM current and x is the offset center beam in horizontal plane. That is true in some approximation when the inter - electrode coupling capacitance is ignored .

The peak output BPM current for harmonic m of the bunch angular frequency ω_0 occurs at t = +/- T/4m and is

$$i_{peak} = 2\langle I_b \rangle\, Z(x, m\omega_0)\, m\omega_0/\beta_b c \times \mathrm{Exp}(-m^2\omega_0^2\sigma^2/2)$$

• What harmonic number is better to use?

To decrease the influence of the possible bunch length variation (3 ÷ 8)ns on output signal it is preferable to use the main harmonic 52.8MHz (m=1). In that case the signal variation will be 1.7dB. For m=2 the signal variation will be 6.6dB.

Remark, if we consider Microwave power sensors as the including many harmonics, the peak output current which occurs at the time t = +/- σ, in time domain representation, varies inversely as square of bunch length RMS

$$i_{peak} = Z(x) \times eN_p / \beta_b c\sigma^2\sqrt{2\pi} \times \mathrm{Exp}(-1/2).$$

As a result the output signal would have a large variation (up to 17dB) at the same condition as before.

• The transfer function $Z(x, \omega_0)$.

To determine the diametrical part of the transfer function we used the expression[/1/]

$$\frac{1}{\pi}\int_0^{\pi} \frac{L(\varphi)(b^2 - r^2)}{b^2 + r^2 - 2br\cos(\varphi - \theta)} d\varphi$$

where r is the displacement from the BPM center, b is the inside electrode radius and φ is the usual polar coordinate. θ is the angle from the median electrode plane to the plane defined by the beam and the BPM center.

The transfer function was calculated analytically for the real pickup electrode design. Also taken into account was the electrodes axes misalignment and inter-electrode coupling capacitance. The estimation was done for the zero beam displacement on $y = r \times \sin\theta$ coordinate ($\theta = 0$) from the center of the BPM.
For the diagonal type electrode $L(\varphi) = L_0\,[1+\cos(\varphi)]/2$.

The transfer functions for hypothetical Right and Left electrodes are:

$$Z_R(x, \omega_0) = L_0[\,1 + K_g \times K_c \times (x + \Delta/2)/b\,]\;/2K_g$$
$$Z_L(x, \omega_0) = L_0[\,1 - K_g \times K_c \times (x - \Delta/2)/b\,]\;/2K_g$$

Where $K_g = (1 + 2f/L_0)^{-1}$ **and**

$$K_c = [R_0\,(2 + 1/R_0\,\omega_0\,C_e)\;\omega_0\,C_e]^{-1}$$

$$(K_t = K_g \times K_c)$$

are the geometrical and the inter-electrodes capacitance factors accordingly. The f and L_0 are the lengths of the flat part and the diagonally inside part of the BPM electrodes, accordingly. The Δ is the electrodes axes misalignment. C_e is the inter-electrodes capacitance and R_0 is the load impedance.

- **The output signal voltage**

  (All calculations were done for beam intensity 1e10ppb)

Table 1. Some BPM and Beam parameters

Parameter	Symbol	Standard BPM	Targeting BPM
Inside BPM radius	b	49.149mm	25.349mm
Diagonal electrode part	L_0	98.298mm	103.947mm
Distance between electrode	d	4.49mm	5 .57mm
Diagonally angle	α	45^0	26^0
Flat electrode part	f	14.351mm	6.968mm
Offset electrodes	Δ	0.4mm	0.2mm
Required resolution	δ_x	0.2mm	0.05mm
Bunch frequency	f_0	52.8MHz	52.8MHz
RMS bunch length	σ	2ns	2ns
Measurable bandwidth	B	1MHz	1MHz
Beam velocity	β	1	1
Inter-electrodes capacitance	C_e	2.97Pf	2.09pF
Geometrical factor	K_g	0.774	0.882

Inter-electrodes capacitance factor	K_c	**0.907**	**0.935**
Total correction factor	K_t	**0.702**	**0.825**
Load impedance	R_0	$50\,\Omega$	$50\,\Omega$

If we use the previous formulas and parameters from Table 1 the peak signal in voltage is given by

$$U_{R(L)} = 9.362 \times 10^{-3} \mathrm{Exp}(-0.055\sigma^2) \times L_0\left[1 + (-)\frac{x + (-)\Delta/2}{b} K_t\right]/2\, K_g$$

The output signals amplitude $U_{R(L)}$ for Standard and Targeting BPM's are presented in Table 2.

- ## The displacement sensitivity and Beam position computing

There are two formulae for displacement sensitivity:

$$(U_R - U_L)/(U_R + U_L) = K_t\,(1 + K_t \times \Delta/2b)^{-1}\, x/b$$

and

$$20 \,\mathrm{Log}_{10}\,(U_R / U_L) = x\, S_x \,.$$

The displacement sensitivity (S_x) is measured in dB/mm.
These parameters for two size BPM's are presented in Table 2.
The wire measurements done by Jim Crisp and the calculated values S_x for standard BPM's are S_x = 0.243 (0.277) dB/mm and S_x = 0.249, respectively.

- ## The position resolution

The position resolution can be derived as

$$\delta(x) = (b / \sqrt{2}\, K_t) \times \delta(U)/U$$

Where $\delta(U)/U$ is the relative accuracy for amplitude voltage measurement. Assuming a required resolution δ_x and the parameters in Table 1 the value $\delta(U)/U$ was specified. The results for both size BPM's are presented in Table 2.

- **The thermal noise and loss budget**

The thermal peak voltage noise was calculated as

$$U_{noise} = \sqrt{2ktBR_0} = 0.64\,\mu V \quad (t=300\,^0K)$$

Translating the thermal voltage noise into a position resolution $\delta(x)$

$$\delta(x)_{noise} = (b/\sqrt{2}\,K_t) \times U_{noise}/U_{R(L)min}$$

For standard BPM's $\delta(x)_{noise} = 0.16\ \mu m$ and for targeting BPM's $\delta(x)_{noise} = 0.06\ \mu m$ for lowest beam intensity(6e9 ppb).
Assuming a required resolution δ_x (Table 1), we have a noise and loss budget 62dB and 58dB accordingly.

- **Dynamic range in beam position and dynamic range available for beam intensity.**

$$x \le [\,2^{N-1}\sqrt{2}\ \delta_x (1+K_t \times \Delta/2b) - \varepsilon\ b/K_t - \Delta/2\,] / (\varepsilon + 2^{N-1}\sqrt{2}\ \delta_x K_t/b)$$

where N is bit digitization number, ε is the beam intensity ratio(dynamic range). Other values are specified in Table 1.
For $\varepsilon = 3.2$ (Beam intensity range 9.5e10ppb to 3e10ppb) and N=12, the dynamic range in beam position is +/- 6.0mm.

Table2.

BPM type	Standard BPM	Targeting BPM
Output signal amplitude $U_{R(L)}$ (Volts)	0.47 (x = 0) 0.61 (x =+20mm) 0.34 (x = -20mm)	0.444 (x = 0) 0.530 (x =+6mm) 0.358 (x = -6mm)
Sensitivity conversion factor S_x (dB/mm)	0.249	0.58
Measurement accuracy δ(U)/U (%)	0.4	0.23

REFERENCES

1. R.Shafer, "Beam Position Monitoring", in *API Conf. Proc.* 212, 26-58(1990)